Carolina Survey Research Laboratory

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# An Assessment of the Audit and the Certification of the Election Result for the November 2012 Election

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May 2013

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GLOBAL PUBLIC HEALTH

This report to North Carolina Board of Elections is on the November 2012 Presidential Election Sample Audit. We begin to present findings from descriptive analyses (based on weighted estimates) of the November 2012 General Election audit data. Next, we provide statistical evidence to quantitatively certify the outcome of the results for the State of North Carolina in 2012.

## A: Findings from Descriptive Analyses of November General Election 2012 Audit Data

Descriptive analyses of the November 2012 audit data followed the same approach that is described in the November 2010 Audit report (Kalsbeek, Zhang & Wang, November 2010), eliminating the discrepancies that are due to human error from the analyses. All of the most recent analyses were completed using a data file of the 205 sample precincts/places, and all findings took into account the type of design this sample turned out to be (i.e., a stratified without-replacement simple random sample of precincts/places, within the 100 NC counties serving as sampling strata). The number of samples selected from each county varied from two precincts/places to four precincts/places based on the magnitude of discrepancies between total election votes and audit votes in each county observed in the May 2012 audit data. All findings again are sample-based estimates of what would have been observed in the state had all 3,222 precincts/places been audited. Since only 205 of the 3,222 precincts/places were actually chosen and observed, these findings were estimates and are subject to sampling error, which was measured but not reported in the tables. The standard error (SE) of most estimates is available in the attached analysis output from SUDAAN. Note that SE findings are also estimates and thus subject to sampling error as well, particularly when sample sizes are less than 50. Most findings are presented for the state as a whole and broken down separately by the type of voting equipment used.

1. *Profile of Candidate Vote Count Discrepancies at the Precinct Level* --- In each sample precinct/place we computed the Discrepancy in Candidate Count (DCC) for the candidate in the precinct/place (i.e., the audit recounts minus the reported election vote counts) to profile the magnitude and direction (greater or less) that the audit counts differed from the election vote counts. A positive discrepancy indicates that the candidate recount was greater than the corresponding candidate election count, while a negative discrepancy indicates that the candidate audit count was lower by that amount. The findings in Table A.1 are population estimates of the percent distribution of the values of DCC. These profiles are presented by the type of voting machine used in the precinct/place.

    - *See Table A.1 and Attachment B.1* --- Contains tabular findings; and the computer printout of the findings from a sample analysis statistical program package called SUDAAN.

**Table A. 1 Percent Distribution of Discrepancies in Candidate Vote Counts (DCC) at the Precinct Level**

| Candidate[a] | Value of DCC -8 | -3 | -2 | 0 | 1 | Precinct/Place Sample Size[b] |
|---|---|---|---|---|---|---|
| | | | **All Machines Combined** | | | 205 |
| 1 | | | | 100.00 | | |
| 2 | | 0.31 | | 99.69 | | |
| 3 | 0.40 | | 0.31 | 98.84 | 0.45 | |
| 4 | | | | 100.0 | | |
| | | | **M100 Machines Only** | | | 165 |
| 1 | | | | 100.00 | | |
| 2 | | | | 100.00 | | |
| 3 | | | | 99.44 | 0.56 | |
| 4 | | | | 100.00 | | |
| | | | **iVotronic Machines Only** | | | 46 |
| 1 | | | | 100.00 | | |
| 2 | | 1.34 | | 98.66 | | |
| 3 | 1.74 | | 1.34 | 96.92 | | |
| 4 | | | | 100.00 | | |

[a]Candidates: 1 = Johnson, 2 = Obama, 3 = Romney, 4 = Write-in.
[b]The sample sizes for M100 and iVotronic are overlapping for 6 precincts/places.

- *Summary of Findings* --- These figures are in estimated percent of all candidate counts with a discrepancy in the amount indicated. Some of the key findings from Table A.1 are:

  a. It is estimated that election candidate counts varied from -8 to +1 vote from those that would be observed in an audit of all votes cast, with more than 90% of candidate counts in the precincts/places having no discrepancy among the four candidates in the race.

  b. Candidate count discrepancies were found in M100 machines, with vote count discrepancies from 0 to +1 vote for M100.

  c. Candidate count discrepancies were also found in iVotronic, showing larger vote count discrepancies than M100 ranging from -8 to 0 votes.

2. *Profile of Total Vote Count Discrepancies at the Precinct Level* --- In each sample precinct/place we used the Discrepancy in Total Count (DTC) for the precinct/place (i.e., the total audit counts minus the total election vote counts) to profile the magnitude and

direction (greater or less) that the total vote audit count for the precinct/place differed from the total election vote count for the same precinct/place. The figures in Table A.2 are population estimates of the percent distribution of the values of DTC. Similarly, a positive discrepancy indicates that the candidate audit count was greater than the corresponding candidate election count, while a negative discrepancy indicates that the candidate audit count was smaller by that amount. These profiles are also presented by the type of voting machine.

- *See Table A.2 and Attachment B.2* --- Contains tabular findings; and the computer printout of the findings from a sample analysis statistical program package called SUDAAN.

**Table A. 2 Percent of Discrepancies in Total Counts (DTC) at the Precinct/Place Level**

| | Value of DTC | | | | Precinct/Place Sample Size* |
|---|---|---|---|---|---|
| | -8 | -5 | 0 | 1 | |
| All Machines Combined | 0.4 | 0.31 | 98.84 | 0.45 | 205 |
| M100 Machine Only | | | 99.44 | 0.56 | 165 |
| iVotronic Machine Only | 1.74 | 1.34 | 96.92 | | 46 |

*The sample sizes for M100 and iVotronic are overlapping for 6 precincts/places.

- *Summary of Findings* --- These figures are in estimated percent of all total precinct/place counts with a discrepancy in the amount indicated. Some of the key findings from Table A.2 are:

  a. Total precinct/place vote count discrepancies, computed as the sum of the corresponding candidate vote count discrepancies for individual precincts/places and as seen in Table A.2, overall varied similarly to the corresponding candidate vote count discrepancies in Table A.1.

  b. It is estimated that total precinct vote count discrepancy among all machines in the state's precincts/places ranged from -8 to +1 vote out of those cast in precincts/places.

  c. Total precinct/place vote count discrepancies for iVotronic machines (-8 to 0 votes) are larger than M100 machines (0 to +1 vote).

3. *Estimated Percent of Precinct Candidate Vote Counts That Are Discrepant* --- We employed the Indicator of Discrepancy for the Candidate Count (IDCC) for the precinct/place to reveal whether or not there was a discrepancy at the precinct level between the audit count and the election count for the candidate. Using values of IDCC for all candidate counts in all sample precincts, we produced the results found in Table A.3, where one finds population estimates of the percent of candidate vote counts with a discrepancy. These percentage rates indicate the chances of a discrepancy in the

candidate vote counts in the election. These rates are presented by the type of voting machine as well.

- *See Table A.3 and Attachment B.3* --- Contains tabular findings and the computer printout of the findings from a sample analysis statistical program package called SUDAAN.

**Table A. 3 Estimated Percent of Discrepant Precinct Candidate Vote Counts**

| Candidate* | All Machines Combined | Precinct/Place Sample Size | M100 Machine Only | Precinct/Place Sample Size | iVotronic Machine Only | Precinct/Place Sample Size |
|---|---|---|---|---|---|---|
| 1 | 0.0000 | 205 | 0.0000 | 165 | 0.0000 | 46 |
| 2 | 0.0031 | 205 | 0.0000 | 165 | 0.0134 | 46 |
| 3 | 0.0116 | 205 | 0.0056 | 165 | 0.0308 | 46 |
| 4 | 0.0000 | 205 | 0.0000 | 165 | 0.0000 | 46 |

*Candidates: 1 = Johnson, 2 = Obama, 3 = Romney, 4 = Write-in.

- *Summary of Findings* --- These figures are in estimated percent of all candidate counts in all precincts/places. Some of the key findings from Table A.3 are:

  a. It is estimated that the percent of discrepant candidate vote counts from all machines varied from 0.00% to 0.0116% among the four candidates.

  b. Candidate vote count discrepancy rates among precincts were higher for the iVotronic machines (0.00% to 0.0308% among candidates) than for the M100 machines (0.00% to 0.0056%).

4. *Estimated Percent of Precinct Total Vote Counts That Are Discrepant* --- Once again we used the Indicator of Discrepancy for the Total Count (IDTC) to reveal whether or not there was a discrepancy between the audit count and the total election count for the precinct/place. Using values of IDTC for all total precinct/place counts in the sample, we produced the results found in Table A.4, where one finds population estimates of the percent of candidate total precinct/place counts with a discrepancy. These percentage rates indicate the chances of a discrepancy in the total precinct/place vote counts in the election. These rates are also presented by the type of voting machine.

   - *See Table A.4 and Attachment B.4* --- Contains tabular findings and the computer printout of the findings from a sample analysis statistical program package called SUDAAN.

**Table A. 4 Estimated Percent of Discrepancies in Total Count at the Precinct/Place Level**

| All Machines Combined | Precinct/Place Sample Size | M100 Machine Only | Precinct/Place Sample Size | iVotronic Machine Only | Precinct/Place Sample Size |
|---|---|---|---|---|---|
| 0.0116 | 205 | 0.0056 | 165 | 0.0308 | 46 |

- *Summary of Findings* --- These figures are in estimated percent of all precincts/places. Some of the key findings from Table A.4 are:

  a. Total election vote count discrepancy rates among precincts/places for all machines combined and for M100 (Table A.4) were similar to corresponding rates among candidates (see Table A.3) because there were only a small amount of variations in election count discrepancy rates observed in the counts of M100 machines.

  b. Total precinct/place audit counts for iVotronic machines were more likely to be discrepant from their election counts (0.0308%) than were audit counts for M100 machines (0.0056%).

5. *Estimated Ratios of Undervotes and Overvotes in the Total Vote Counts Statewide* --- This statistical measurement we computed was used to compare the accuracy of voting machines: M100 and iVotronic. We used two measures, that is, undervotes and overvotes in the total counts statewide to assess the accuracy. "Undervotes" are defined as the negative differences between the election counts and the audit counts when subtracting audit counts from election counts and "overvotes" are the positive differences when subtracting audit counts from election counts. Using values of the undervotes and overvotes for all total precincts/places, we produced the results found in Table A.5, where one finds population estimates of the proportion of undervotes/overvotes contributed by M100 machines and iVotronic machines in the election. These ratio estimates indicated that the percentage of the total votes was undercounted or over-counted.

   - *See Table A.5 and Attachment B.5* --- Contains tabular findings and the computer printout of the findings from a sample analysis statistical program package called SUDAAN.

**Table A. 5 Ratio Estimates for Undervotes/Overvotes in M100 and iVotronic and Total Votes**

| Voting machine | Sample size* | Ratio estimate | Standard Error | T-Test | *p*-value |
|---|---|---|---|---|---|
| Undervote | | | | 1.00 | 0.32 |
| M100 | 165 | 0.00001 | 0.00001 | | |
| iVotronic | 46 | 0.00000 | 0.00000 | | |
| Total | 205 | 0.00000 | 0.00000 | | |
| Overvote | | | | -1.33 | 0.75 |
| M100 | 165 | 0.00000 | 0.00000 | | |
| iVotronic | 46 | 0.00020 | 0.00015 | | |
| Total | 205 | 0.00005 | 0.00003 | | |

**Note.* The sample sizes for M100 and iVotronic are overlapping for 6 precincts/places.

- *Summary of Findings* --- These figures are in estimated ratios of all precincts/places. "Student's" t-test was conducted to compare if M100 machines performed statistically significantly different from iVotronic machines with regard to overvotes and undervotes. Some of the key findings from Table A.5 are:

    a. M100 machines did not differ from iVotronic with regard to undercounting the election, which is indicated by their estimates ($t = 1.00, p = 0.32$).

    b. M100 machines and iVotronic machines were not likely to over-count the election votes, which is indicated by the non-significant t-value ($t = -1.33, p = 0.75$).

*Comparison between the November 2012 Sample Findings and the May 2012 Sample Findings*

Given that the same analytic strategy was applied to the May and November Audit data, which solely considered the machine-error discrepancies, the two data sets showed a similar trend. Variations between the audit votes and the election votes across candidates were limited in the May sample (only -2 to 0 votes), while slightly larger in the November sample (-8 to +1 vote). A similar pattern was observed between the total audit votes and the election votes in these two samples. Over 99% of candidate votes in the May sample indicated no discrepancies, 97% in the November sample.

The population estimates of the percent of candidate vote counts (i.e., IDCC) with a discrepancy in the May 2012 sample varied from 0.000% to 0.007% across the candidates (Table A.3), while in the November 2012 sample, they were 0.000% to 0.0116%. The discrepancies of estimated percentage of candidate total precinct/place counts (i.e., IDTC, see Table A.4) were similar in in the May 2012 sample (0.01%) and the November 2010 sample (0.0116%).

Finally, M100 machines showed higher tally agreement percentage in total counts at the precinct/place level than iVotronic machines in both samples (99.26% vs. 98.6% in the May sample and 99.44% vs. 96.92% in the November sample). By eliminating the human-error-related discrepancies from the sample analyses, the performance accuracy of M100 has been consistently high.

## B: Election Certification from the November 2012 Sample Audits

Following methodology we recently developed for use with sample audit data starting with the November 2012 primary election [see Attachment A], we are able to quantitatively certify the outcome of elections in the State of North Carolina. The measure we obtain in using our methodology is the statistical probability that the declared "Winner" on Election Day in fact defeated the Election Day "Loser". This result is obtained from the reported election results for the state as a whole and the results of a manual audit recount in a random sample of the State's

"precincts"[1] with the manual count presumed to be the "true" count of votes. The "Election Day" "Winner" and "Loser" defined here are the "Winner" and "Loser", respectively, based on the statewide vote count on Election Day. Also, we define the "actual" "Winner" and "Loser" as the "Winner" and "Loser", respectively, based on a projected manual vote audit in all precincts in the state.

If this probability exceeded some reasonable threshold, the election results can be considered certified. If, for example, the certification threshold is set at 99.9%, then another way of interpreting a certified election is that there is less than a 1 in 1,000 chance that the declared "Loser" based on the Election Day results should have been declared the (actual) "Winner". While we will arbitrarily adopt 99.9% as the threshold in this report, clearly there should be some consideration by the North Carolina Board of Elections as to what the threshold should be for future election audits.

Since a manual audit is in reality only available in the randomly selected precincts, actual statewide counts for the candidates are unknown, but they can be estimated from the audit sample since random selection is used to choose the sample. The approach for doing this is now briefly summarized based on Attachment A.

1. *Calculate the probability that the Election Day "Winner" is the actual "Winner":* In each sample precinct we used the discrepancy between the Election Day count and audit recount for the "Winner" to profile the magnitude and direction that the audit recount for the precinct differed from the reported vote count for the "Winner" for the same precinct. A positive discrepancy indicates that the audit recount is greater than the Election Day count, while a negative discrepancy indicates the Election Day count is greater than the audit recount.[2] The mean discrepancy among all of the State's precincts is then estimated from the sample precincts. We refer to this estimate as $\overline{d}_W$. To determine the probability that the Election Day "Winner" is in fact the actual "Winner", we need to know how large the average discrepancy must be among all precincts in the state to overturn the Election Day results for the "Winner" and "Loser". We refer to this outcome-altering average discrepancy amount as $\overline{D}_0$. Assuming that $\overline{d}_W$ follows *"Student's"* t-distribution among all possible precinct samples that we could have chosen, the probability that the Election Day "Winner" is the actual "Winner" is,

   $\pi_W = \Pr\{\overline{d}_W^* \leq \frac{\overline{D}_0 - \overline{d}_W}{se(\overline{d}_W)}\}$, where $se(\overline{d}_W)$ is the standard error of $\overline{d}_W$, which measures how variable $\overline{d}_W$ is among all possible precinct samples that could have been chosen.

   - *See Attachment C.1*---Contains tabular findings, followed immediately by the computer printout of the findings from a sample analysis and statistical analysis program package called SAS-callable-SUDAAN.

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[1] Voting places in North Carolina include both "precincts," where people go to vote on Election Day, as well as other "places" where they can return ballots cast prior to Election Day. For simplicity, we refer to all both types of ballot assembly points here as "precincts."

[2] Notice that "discrepancy" between Election Day reported and audit recount numbers in this section is different from the way "discrepancy" was defined in the previous descriptive findings. Discrepancy in this section = Audit Recount – Reported, while Discrepancy in the descriptive findings = Reported – Audit Recount.

- *Summary of findings*---Table B.1 shows the estimated $\overline{d}_W$, standard error of $\overline{d}_W$ (se( $\overline{d}_W$ )), $\overline{D}_0$, $\pi_W$ and the probability that the Election Day "Loser" defeated "Winner". The key findings are:

  a. Compared to $\overline{D}_0$, $\overline{d}_W$ is relatively small, thus the probability that the Election Day "Winner" defeated "Loser" approaches to 0.99999+, and the probability that Election Day "Loser" defeated "Winner" approaches to 0, which clearly shows that the Election Day "Winner" is the actual "Winner".

  b. Using three different options to compute $\overline{D}_0$, we obtained similar values for probability $\pi_W$.

**Table B.1 Probability for the Election Day "Winner" Defeated the "Loser"**

| Different Options in Computing $\overline{D}_0$ | Estimated Mean Discrepancy ($\overline{d}_W$) | Standard Error for Estimated Mean Discrepancy ($se(\overline{d}_W)$) | Value for the Election Day "Loser" to be Actual "Winner" ($\overline{D}_0$) | Probability Election Day "Loser" Is the Actual "Winner"* ($1-\pi_W$) | Probability Election Day "Winner" Is the Actual "Winner"* ($\pi_W$) |
|---|---|---|---|---|---|
| 1 | -0.0711 | 0.1167 | 14.2772 | $4\times 10^{-112}$ | 0.9999+ |
| 2 | -0.0711 | 0.1167 | 1.3821 | $1.98\times 10^{-22}$ | 0.9999+ |
| 3 | -0.0711 | 0.1167 | 355.8367 | $7.9\times 10^{-253}$ | 0.9999+ |

We can conclude from these results that the statistical probability that "Winner" fact defeated "Loser" the North Carolina general election is 0.99+.

2. *The 95% confidence interval for estimated total actual vote for the Election Day "Winner" and "Loser":* Since manual audit is only available in selected precincts, the total actual vote count is unknown. However, we can estimate this vote count and provide a 95% confidence interval for this actual total vote count. If the reported total vote count is within this 95% confidence interval, we then can claim the reported total vote count is correct and can represent the actual total vote count. Based on the total number of audit for the "Winner" and "Loser" in selected precincts, we are able to calculate an estimate for the total number of audit for the whole state and the estimated standard error of this point estimate for "Winner" and "Loser" respectively by using *proc ratio* in SUDAAN. Then the 95% confidence interval for total actual vote count is computed using the point estimate and its estimated standard error by SAS.

   - *See Attachment C.2*---Contains tabular findings and graph, followed immediately by the computer printout of the findings from a sample analysis and statistical analysis program package called SAS-callable-SUDAAN.

   - *Summary of findings*---Table B.2 shows the estimated total audit recount and its 95% confidence interval for all precincts statewide. The key findings are:

a. The estimated total audit numbers are very close to the reported vote count for the "Winner" and the "Loser" respectively.

b. The estimated total audit number for the "Winner" is bigger than the number for the "Loser".

c. The reported total counts for "Winner" and "Loser" are within the computed 95% confidence intervals of estimated total audit respectively. Thus the reported total count may be accurate based on audit data. Notice the fact that the estimated total audit for the Election Day "Winner" is higher than the "Loser", we can certify the result of this election and claim that Romney is the actual "Winner".

**Table B.2 95% Confidence Interval for Actual Total Vote Audit in "Winner" and "Loser"**

| | Reported Total Count | Estimated Total Audit | 95% Confidence Interval For Total Audit | |
|---|---|---|---|---|
| | | | Lower | Upper |
| Winner | 2270395 | 2308474 | 2295227 | 2321721 |
| Loser | 2178391 | 2223438 | 2197817 | 2249058 |

Reference:

Kalsbeek, W., Zhang, L., & Wang, X. (November 2010). An Assessment of the Audit and Certification of the Election Result for the November 2010 General Election. Work report to North Carolina State Board of Elections.

# Attachment A

## Certification Methodology for North Carolina Elections

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March 2, 2010

## Background

The North Carolina Board of Elections conducts primary elections in May and a general election in November on even numbered years. In 2005 the North Carolina Legislature passed a law requiring: (i) that an audit of the vote count for statewide races be conducted immediately after the election, and (ii) that the audit be done on a random sample of precincts.[3] The legislation also specifies only that at least one precinct must be selected in each of North Carolina's 100 counties. There is no indication as to how, or for what purpose, the audit sample data are to be analyzed, however.

The Survey Research Unit has a standing agreement with the NCBOE to design and choose the sample of precincts/places for the audit for each election. The SRU (by default) has also been given the task of analyzing the data from the audit sample. The sample for the first audit was a stratified simple random sample of 200 precincts/places, with two precincts/places selected in each county, which served as strata.[4] The sample size for each county stratum in the November 2010 general election was determined by the size of the discrepancy in total count (DTC) in the November 2010 sample of precincts for all candidates in the vote for the statewide office designated for audit in that county. Specifically, the number of sampled precincts in the November 2010 election audit for any county was two plus the sum of the magnitude (i.e., absolute value) of the DTCs for the May 2006 sample precincts in the county. This rule led to a November 2010 audit sample size of 201 precincts statewide.

With no directive on what to analyze in these two election audit samples, we simply profiled the size of discrepancies between reported and audit counts for individual candidates and all candidates combined. We also compared the size of discrepancies between the two types of voting machines used in the state (i.e., M100 and iVotronic) and found in both election samples that one (the M100) generally had larger discrepancies.

## Objective

In addition to randomly selecting a sample of precincts for the audit recount of designated statewide races in North Carolina elections, and producing simple comparative profiles of discrepancies between the reported vote counts and the audit recounts, the SRU has agreed to develop a quantitative measure to aid in "certifying" the outcome of designated races in each election. This measure should be based solely on the reported election results in all precincts of the state and on the manual audit results for a random sample of precincts that is chosen by SRU staff the day after the election votes have been reported. Since the SRU is not at all involved in the audit process, we presume: (i) that all statutory requirements are met in completing the audit and (ii) that the audit results can be considered to be the best available "actual" count of the votes (i.e., the audit results are assumed to be error-free).

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[3] A race in any election is consider "statewide" if voters in the state can vote for the candidates running in it. A "precinct" refers to a place where voters go to vote on Election Day or where absentee ballots can be returned in advance of the election. There were 3,015 precincts established for use in the May 2008 primary election.

[4] Stratification by county is the only way to assure that at least one precinct is selected in each county, and two is the minimum stratum size needed to measure the statistical precision of estimates from the sample.

The methodology we have developed presumes that there are $K \geq 2$ candidates in each designated statewide race. It focuses exclusively on the top two finishers in the designated race. We consider the candidate with the top votes in the race to be the "winner". And the candidate with the second number of votes to be the "loser." The winner and loser based on the total statewide reported votes for each candidate on Election Day will be defined as the "apparent" winner and loser, respectively, while the winner and loser based on a (hypothetical) complete manual vote audit in all precincts in the state is defined as the "rightful" winner and loser, respectively. Since the manual audit will only be available for a random sample of precincts, the rightful winner and loser are unknown but can be estimated from the audit sample. The authenticity measure proposed here for each race is the probability that the apparent winner in fact defeated the apparent loser. We describe the approach in detail after defining the notation needed to describe it.

**Notation**

The table below defines the various terms that are used in the approach:

| **Symbol** | **Definition** |
|---|---|
| $C$ | Number of counties with at least one precinct in the race; these counties are sampling strata since by law an independent random sample (we use SRS) of precincts/places is chosen in each county $(C = 100$ in North Carolina$)$ |
| $c$ | The integer subscripts to designate county $(c = 1,2,\ldots,C)$ |
| $N_c$ | Number of precincts/places in the race in county $c$ |
| $N = \sum_{c=1}^{C} N_c$ | Total number of precincts/places in the state (e.g., 3,035 for the November 2006 general election) |
| $n_c$ | Number of precincts/places selected by SRS in county $c$ |
| $n = \sum_{c=1}^{C} n_c$ | Total sample size of precincts/places in the state (e.g., 264 for the November 2006 general election audit sample) |
| $v_{kp}$ ; $v_{Wp}$; $v_{Lp}$ | Election Day vote count for candidate $k$ in precinct $p$; for $k=W$ (the Election Day winner); for $k=L$ (the Election Day loser) |
| **Notation** | **Definition** |

| | |
|---|---|
| $V_W$ , $V_L$ | Total reported vote for the Election Day winner ($V_W$ ); and total reported vote for the Election Day loser ( $V_L$ ) [5] |
| $V = V_W + V_L$ | Total reported vote combined for the winner and loser |
| $a_{kp}$ ; $a_{Wp}$; $a_{Lp}$ | Actual audit vote for candidate $k$ in precinct $p$; for $k=W$ (the Election Day winner); for $k=L$ (the Election Day loser) |
| $A_W$ , $A_L$ | Total "actual" vote for the Election Day winner ( $A_W$ ); and total reported vote for the Election Day loser ( $A_L$ ) |
| $A = A_W + A_L$ | Total "actual" vote combined for the winner and loser |
| $d_{Wp} = a_{Wp} - v_{Wp}$ | Discrepancy between actual (audit) vote count and the Election Day vote count for the Election Day winner in precinct $p$ |
| $D_W = \sum_{p=1}^{N} d_{Wp}$ | Total discrepancy among all $N$ precincts/places in the state |
| $\bar{D}_W = \sum_{p=1}^{N} d_{Wp} / N = D_W / N$ | Mean discrepancy among all $N$ precincts/places in the state |
| $\omega_{cp} = N_c / n_c$ | Sample weight for all $n_c$ sample precincts/places in county $c$ |
| $\bar{d}_W$ | Usual stratified SRS estimator of a population mean (of $d_{Wp}$ ) |
| $se(\bar{d}_W)$ | Usual stratified SRS estimator of the standard error of an estimated population mean (of $d_{Wp}$ ) |

[5] In races with > 2 candidates (see last section), the "loser" will be the 2nd place finisher and the test will be for the correct placement of the 1st and 2nd place finishers in the race.

**Approach**

We use $v_{kp}$ to denote the vote count reported for candidate $k$ in precinct $p$, and $a_{kp}$ denotes the actual vote count for candidate $k$ in precinct $p$ if a manual audit recount was conducted there. $V = V_W + V_L$ is the total combined reported vote count for the apparent winner and loser in the state's $N$ precincts, where $V_W = \sum_{p=1}^{N} v_{Wp}$ and $V_L = \sum_{p=1}^{N} v_{Lp}$ are the statewide reported vote counts for the apparent winner and loser, respectively. If an audit recount were conducted statewide to determine the actual vote count for all candidates in the $N$ precincts, $A = A_W + A_L$ would be the total combined statewide actual vote count for the apparent winner and loser, where $A_W = \sum_{p=1}^{N} a_{Wp}$ and $A_L = \sum_{p=1}^{N} a_{Lp}$ are the statewide actual vote counts for the apparent winner and loser, respectively. Finally, define $d_{Wp} = a_{Wp} - v_{Wp}$ as the discrepancy between the reported and actual vote count for the apparent winner so that $D_W = \sum_{p=1}^{N} d_{Wp} = V_W - A_W$ is the total statewide discrepancy between reported and actual vote counts for the apparent winner and $\bar{D}_W = \sum_{p=1}^{N} d_{Wp} / N = D_W / N$ is the mean discrepancy among precincts in the state.

We are interested in the event ($\xi_{WW}$) that the apparent winner is the rightful winner of the statewide election. If $A$ were known, $\xi_{WW}$ will occur if the sum of discrepancies in all precincts ( $D_W$ ) is no greater than the difference between the statewide vote count for the apparent winner ( $V_W$ ) and the minimum actual statewide vote count the apparent winner must have to be considered the rightful winner of the election (i.e., $Int\{A^*/2\}+1$, where in general $Int\{X\}$ is the integer portion of the numerical value $X$) and $A^*$ is a suitable proxy value for $A$ (see options below). Thus, we might test the null hypothesis, $H_o : D_W \le D_o$ where $D_o = V_W - [Int\{A^*/2\}+1]$, against the alternative hypothesis, $H_A : D_W > D_o$; or equivalently we can test $H_o : \bar{D}_W \le \bar{D}_o$ and $H_A : \bar{D}_W > \bar{D}_o$, where $\bar{D}_o = \left[V_W - [Int\{A^*/2\}+1]\right]/N$.

A second approach is to estimate $\bar{D}_W$ from the sample of $n$ audit precincts and to determine the probability that $\bar{D}_W$ will be no greater than the value that would make the apparent loser the rightful winner. Once again a proxy value for $A$ (i.e., $A^*$) is needed. Using standard formulae for estimation from a stratified simple random sample, the estimator of $\bar{D}_W$ will be,

$$\bar{d}_W = \sum_{c=1}^{C}(N_c / N)\bar{d}_{Wc} = \frac{\sum_{c=1}^{C}\sum_{p=1}^{n_c}(N_c / n_c)d_{Wcp}}{N} = \frac{\sum_{c=1}^{C}\sum_{p=1}^{n_c}(N_c / n_c)d_{Wcp}}{\sum_{c=1}^{C}\sum_{p=1}^{n_c}(N_c / n_c)} \quad , \qquad [1]$$

where the two ratio expressions are the usual weighted estimator of a mean in SUDAAN *proc descript*, with $N_c / n_c$ as the weight for each sample member, *design=strwor* as the design option in the proc statement, and the created variable, the *nest* statement as *nest county*; with the variable, *county*, as the stratum identifier for the sample precinct, and *N_c*, taking values $N_c$ depending on the county *(c)* from which the sample precinct was selected for the *totcnt* statement. The corresponding estimated variance of $\bar{d}_W$ is compute by this same SUDAAN setup as,

$$[se(\bar{d}_W)]^2 = \sum_{c=1}^{C}\left(\frac{N_c}{N}\right)^2 \frac{1-n_c/N_c}{n_c}\, s_{W_c}^2 = \sum_{c=1}^{C}\left(\frac{N_c}{N}\right)^2 \frac{1-n_c/N_c}{n_c}\, \frac{\sum_{p=1}^{n_c}(d_{Wcp}-\bar{d}_{Wc})^2}{n_c-1} \qquad [2]$$

If we can assume that $\bar{d}_W$ follows a *"Student's" t-distribution* with *n-C* degrees of freedom with mean, $\bar{D}_W$, and variance, $V(\bar{d}_W)$, then another direct certification indicator for the statewide race that is audited is the following probability that the apparent winner in fact defeated the apparent loser; i.e.,

$$\pi_W \equiv Pr\left\{\bar{d}_W \le \bar{D}_o\right\} = Pr\left\{\bar{d}_W \le \frac{\{V_W - [Int\{A^*/2\}+1]\}}{N}\right\} , \qquad [3]$$

where $A^*$ is a proxy measure of *A*. Obtaining the expression in Eq. [3] requires a transformation of $\bar{d}_W$ (referred to as $\bar{d}_W^*$) that will follows a standard *t-distribution* with mean zero and unit variance. The probability that the apparent winner in fact defeated the apparent loser, as obtained from $\bar{d}_W^*$, will thus be,

$$\pi_W \equiv Pr\left\{\bar{d}_W^* \le \frac{\bar{D}_o - \bar{d}_W}{se(\bar{d}_W)}\right\} = Pr\left\{\bar{d}_W^* \le \frac{\{V_W - [Int\{A^*/2\}+1]\}/N - \bar{d}_W}{se(\bar{d}_W)}\right\} . \qquad [4]$$

Both of the above approaches require a measure for *A*, which is unknown. The following are optional values for $A^*$:

1. $A^* = V$ --- This option assumes that A=V, which assumes that there are no overvotes (commission errors in reported vote count) or undervotes (omission errors in reported vote count) and that any misappropriation of votes to the winner is from the loser, and vice versa. This assumption is risky.

2. $A^* = \hat{A}_c$ --- $\hat{A}_c = V\hat{R}_c = V\frac{\hat{A}}{\hat{V}}$ is a combined estimator of $A$ obtained from the stratified SRS of precincts. One way to compute $\hat{A}_c$ and its estimated standard error, $se(\hat{A}_c) = V\{se(\hat{R}_c)\}$, is to obtain $\hat{R}_c$ and $se(\hat{R}_c)$ from *proc ratio* in SUDAAN using the total actual winner and loser vote count for each precinct (i.e., $a_p = a_{Wp} + a_{Lp}$), as the numerator variable, the total reported winner and loser vote count from the election for each precinct (i.e., $v_p = v_{Wp} + v_{Lp}$), as the denominator variable, and the same design setup as used to obtain $\bar{d}_W$ and its standard error (see text immediately following Eq. [1]). Since $V$ is known, the estimates of $\hat{A}$ and $se(\hat{A}_c)$ follow directly from the fact that $\hat{A}_c = V\hat{R}_c$ and $se(\hat{A}_c) = V\{se(\hat{R}_c)\}$.

3. $A^* = \hat{A} - 1.96\{se(\hat{A})\}$ --- Using the lower bound of a 95% confidence interval for $A$ is almost certain to produce the most conservative value for $\pi_W$.

Using Eq. [4], produce $\pi_W$ the probability that the apparent winner in fact defeated the apparent loser using all three options for $A^*$.

# Attachment B.1

# Attachment B.2

# Attachment B.3

# Attachment B.4

# Attachment B.5

# Attachment B.1



S U D A A N
Software for the Statistical Analysis of Correlated Data

Release 11.0.0

DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method, Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID
Stage 1 Sample Count Variable: SAMPLESIZENOV12

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105



Frequencies and Values for CLASS Variables
by: Total diff @candidate1.

| Total diff @candidate1 | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 191 | 0 |



Frequencies and Values for CLASS Variables

by: Total diff @candidate2.

| Total diff @candidat-e2 | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 1 | -3 |
| Ordered Position: 2 | 204 | 0 |



Frequencies and Values for CLASS Variables
by: Total diff @candidate3.

| Total diff @candidat-e3 | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 1 | -8 |
| Ordered Position: 2 | 1 | -2 |
| Ordered Position: 3 | 202 | 0 |
| Ordered Position: 4 | 1 | 1 |



Frequencies and Values for CLASS Variables
by: Total diff @candidate4.

| Total diff @candidat-e4 | Frequency | Value |
|---|---|---|

```
Ordered
  Position:
  1                   117        0
---------------------------------
```



```
Frequencies and Values for CLASS Variables
by: M100 diff @candidate1.

---------------------------------
M100 diff
  @candidat-
  e1              Frequency    Value
---------------------------------
Ordered
  Position:
  1                   152        0
---------------------------------
```



```
Frequencies and Values for CLASS Variables
by: M100 diff @candidate2.

---------------------------------
M100 diff
  @candidat-
  e2              Frequency    Value
---------------------------------
Ordered
  Position:
  1                   165        0
---------------------------------
```



```
Frequencies and Values for CLASS Variables
by: M100 diff @candidate3.

---------------------------------
M100 diff
  @candidat-
  e3              Frequency    Value
---------------------------------
```

```
Ordered
  Position:
  1                   164        0
Ordered
  Position:
  2                     1        1
---------------------------------
```



Frequencies and Values for CLASS Variables
by: M100 diff @candidate4.

```
---------------------------------
M100 diff
  @candidat-
  e4            Frequency    Value
---------------------------------
Ordered
  Position:
  1                    89        0
---------------------------------
```



Frequencies and Values for CLASS Variables
by: iVo diff @candidate1.

```
---------------------------------
iVo diff
  @candidat-
  e1            Frequency    Value
---------------------------------
Ordered
  Position:
  1                    45        0
---------------------------------
```



Frequencies and Values for CLASS Variables
by: iVo diff @candidate2.

```
---------------------------------
iVo diff
```

| @candidat-e2 | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 1 | -3 |
| Ordered Position: 2 | 45 | 0 |



Frequencies and Values for CLASS Variables
by: iVo diff @candidate3.

iVo diff

| @candidat-e3 | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 1 | -8 |
| Ordered Position: 2 | 1 | -2 |
| Ordered Position: 3 | 44 | 0 |



Frequencies and Values for CLASS Variables
by: iVo diff @candidate4.

iVo diff

| @candidat-e4 | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 30 | 0 |

Variance Estimation Method: Taylor Series (STRWOR)
by: Total diff @candidate1.

| Total diff @candidate1 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 191 | 3083.50 | 100.00 | 0.00 | . | . |
| 0 | 191 | 3083.50 | 100.00 | 0.00 | . | . |



Variance Estimation Method: Taylor Series (STRWOR)
by: Total diff @candidate2.

| Total diff @candidate2 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 205 | 3222.00 | 100.00 | 0.00 | . | . |
| -3 | 1 | 10.00 | 0.31 | 0.29 | 0.05 | 2.01 |
| 0 | 204 | 3212.00 | 99.69 | 0.29 | 97.99 | 99.95 |



Variance Estimation Method: Taylor Series (STRWOR)
by: Total diff @candidate3.

| Total diff @candidate3 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 205 | 3222.00 | 100.00 | 0.00 | . | . |
| -8 | 1 | 13.00 | 0.40 | 0.39 | 0.06 | 2.67 |
| -2 | 1 | 10.00 | 0.31 | 0.29 | 0.05 | 2.01 |
| 0 | 202 | 3184.50 | 98.84 | 0.65 | 96.50 | 99.62 |
| 1 | 1 | 14.50 | 0.45 | 0.43 | 0.07 | 3.00 |

Date: 05-25-2013 SUDAAN Page: 16
Time: 20:14:21 Table: 4

Variance Estimation Method: Taylor Series (STRWOR)
by: Total diff @candidate4.

| Total diff @candidate4 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 117 | 2006.00 | 100.00 | 0.00 | . | . |
| 0 | 117 | 2006.00 | 100.00 | 0.00 | . | . |

Date: 05-25-2013 SUDAAN Page: 17
Time: 20:14:21 Table: 5

Variance Estimation Method: Taylor Series (STRWOR)
by: M100 diff @candidate1.

| M100 diff @candidate1 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 152 | 2472.50 | 100.00 | 0.00 | . | . |
| 0 | 152 | 2472.50 | 100.00 | 0.00 | . | . |

Date: 05-25-2013 SUDAAN Page: 18
Time: 20:14:21 Table: 6

Variance Estimation Method: Taylor Series (STRWOR)
by: M100 diff @candidate2.

| M100 diff @candidate2 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 165 | 2597.50 | 100.00 | 0.00 | . | . |
| 0 | 165 | 2597.50 | 100.00 | 0.00 | . | . |

Date: 05-25-2013 SUDAAN Page: 19
Time: 20:14:21 Table: 7

Variance Estimation Method: Taylor Series (STRWOR)
by: M100 diff @candidate3.

| M100 diff @candidate3 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 165 | 2597.50 | 100.00 | 0.00 | . | . |
| 0 | 164 | 2583.00 | 99.44 | 0.54 | 96.30 | 99.92 |
| 1 | 1 | 14.50 | 0.56 | 0.54 | 0.08 | 3.70 |

Date: 05-25-2013 SUDAAN Page: 20
Time: 20:14:21 Table: 8

Variance Estimation Method: Taylor Series (STRWOR)
by: M100 diff @candidate4.

| M100 diff @candidate4 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 89 | 1500.50 | 100.00 | 0.00 | . | . |
| 0 | 89 | 1500.50 | 100.00 | 0.00 | . | . |

Date: 05-25-2013 SUDAAN Page: 21
Time: 20:14:21 Table: 9

Variance Estimation Method: Taylor Series (STRWOR)
by: iVo diff @candidate1.

| iVo diff @candidate1 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 45 | 734.00 | 100.00 | 0.00 | . | . |

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 45 | 734.00 | 100.00 | 0.00 | . | . |



Variance Estimation Method: Taylor Series (STRWOR)
by: iVo diff @candidate2.

| iVo diff @candidate2 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 46 | 747.50 | 100.00 | 0.00 | . | . |
| -3 | 1 | 10.00 | 1.34 | 1.27 | 0.20 | 8.37 |
| 0 | 45 | 737.50 | 98.66 | 1.27 | 91.63 | 99.80 |



Variance Estimation Method: Taylor Series (STRWOR)
by: iVo diff @candidate3.

| iVo diff @candidate3 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 46 | 747.50 | 100.00 | 0.00 | . | . |
| -8 | 1 | 13.00 | 1.74 | 1.67 | 0.25 | 10.96 |
| -2 | 1 | 10.00 | 1.34 | 1.27 | 0.20 | 8.37 |
| 0 | 44 | 724.50 | 96.92 | 2.10 | 88.63 | 99.22 |



Variance Estimation Method: Taylor Series (STRWOR)
by: iVo diff @candidate4.

| iVo diff @candidate4 | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| Total | 30 | 522.50 | 100.00 | 0.00 | . | . |
| 0 | 30 | 522.50 | 100.00 | 0.00 | . | . |

# Attachment B.2



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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method, Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID
Stage 1 Sample Count Variable: SAMPLESIZENOV12

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105



Frequencies and Values for CLASS Variables
by: Total diff @ precin.

| Total diff @ precin | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 1 | -8 |
| Ordered Position: 2 | 1 | -5 |
| Ordered Position: 3 | 202 | 0 |
| Ordered Position: 4 | 1 | 1 |

Frequencies and Values for CLASS Variables
by: M100 diff @ precin.

| M100 diff @ precin | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 164 | 0 |
| Ordered Position: 2 | 1 | 1 |



Frequencies and Values for CLASS Variables
by: iVo diff @ precin.

| iVo diff @ precin | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 1 | -8 |
| Ordered Position: 2 | 1 | -5 |
| Ordered Position: 3 | 44 | 0 |



Variance Estimation Method: Taylor Series (STRWOR)
by: Total diff @ precin.

| | Lower | Upper |
|---|---|---|

Total diff @ precin

| | Sample Size | Weighted Size | Row Percent | SE Row Percent | 95% Limit ROWPER | 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 205 | 3222.00 | 100.00 | 0.00 | . | . |
| -8 | 1 | 13.00 | 0.40 | 0.39 | 0.06 | 2.67 |
| -5 | 1 | 10.00 | 0.31 | 0.29 | 0.05 | 2.01 |
| 0 | 202 | 3184.50 | 98.84 | 0.65 | 96.50 | 99.62 |
| 1 | 1 | 14.50 | 0.45 | 0.43 | 0.07 | 3.00 |



Variance Estimation Method: Taylor Series (STRWOR)
by: M100 diff @ precin.

M100 diff @ precin

| | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 165 | 2597.50 | 100.00 | 0.00 | . | . |
| 0 | 164 | 2583.00 | 99.44 | 0.54 | 96.30 | 99.92 |
| 1 | 1 | 14.50 | 0.56 | 0.54 | 0.08 | 3.70 |



Variance Estimation Method: Taylor Series (STRWOR)
by: iVo diff @ precin.

iVo diff @ precin

| | Sample Size | Weighted Size | Row Percent | SE Row Percent | Lower 95% Limit ROWPER | Upper 95% Limit ROWPER |
|---|---|---|---|---|---|---|
| Total | 46 | 747.50 | 100.00 | 0.00 | . | . |
| -8 | 1 | 13.00 | 1.74 | 1.67 | 0.25 | 10.96 |
| -5 | 1 | 10.00 | 1.34 | 1.27 | 0.20 | 8.37 |
| 0 | 44 | 724.50 | 96.92 | 2.10 | 88.63 | 99.22 |

# Attachment B.3



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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method, Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105

Date: 05-25-2013 SUDAAN Page: 1
Time: 20:15:22 Table: 1

Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| Total dummy-diff @candidate1 1=yes 0=no | Sample Size | 191.0000 | 191.0000 |
| | Weighted Size | 3083.5000 | 3083.5000 |
| | Total | 0.0000 | 0.0000 |
| | Lower 95% Limit Total | 0.0000 | 0.0000 |
| | Upper 95% Limit Total | 0.0000 | 0.0000 |
| | Mean | 0.0000 | 0.0000 |
| | SE Mean | 0.0000 | 0.0000 |
| | Lower 95% Limit Mean | 0.0000 | 0.0000 |
| | Upper 95% Limit Mean | 0.0000 | 0.0000 |
| Total dummy- | Sample Size | 205.0000 | 205.0000 |

| | | | |
|---|---|---|---|
| diff | Weighted Size | 3222.0000 | 3222.0000 |
| @candidate2 | Total | 10.0000 | 10.0000 |
| 1=yes 0=no | Lower 95% Limit | | |
| | Total | -8.8106 | -8.8106 |
| | Upper 95% Limit | | |
| | Total | 28.8106 | 28.8106 |
| | Mean | 0.0031 | 0.0031 |
| | SE Mean | 0.0029 | 0.0029 |
| | Lower 95% Limit | | |
| | Mean | -0.0027 | -0.0027 |
| | Upper 95% Limit | | |
| | Mean | 0.0089 | 0.0089 |



Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| Total dummy-diff @candidate3 1=yes 0=no | Sample Size | 205.0000 | 205.0000 |
| | Weighted Size | 3222.0000 | 3222.0000 |
| | Total | 37.5000 | 37.5000 |
| | Lower 95% Limit | | |
| | Total | -4.1745 | -4.1745 |
| | Upper 95% Limit | | |
| | Total | 79.1745 | 79.1745 |
| | Mean | 0.0116 | 0.0116 |
| | SE Mean | 0.0065 | 0.0065 |
| | Lower 95% Limit | | |
| | Mean | -0.0013 | -0.0013 |
| | Upper 95% Limit | | |
| | Mean | 0.0246 | 0.0246 |
| Total dummy-diff @candidate4 1=yes 0=no | Sample Size | 117.0000 | 117.0000 |
| | Weighted Size | 2006.0000 | 2006.0000 |
| | Total | 0.0000 | 0.0000 |
| | Lower 95% Limit | | |
| | Total | 0.0000 | 0.0000 |
| | Upper 95% Limit | | |
| | Total | 0.0000 | 0.0000 |
| | Mean | 0.0000 | 0.0000 |
| | SE Mean | 0.0000 | 0.0000 |
| | Lower 95% Limit | | |
| | Mean | 0.0000 | 0.0000 |
| | Upper 95% Limit | | |

| | | | |
|---|---|---|---|
| | Mean | 0.0000 | 0.0000 |



Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| M100 dummy-diff @candidate1 1=yes 0=no | Sample Size | 152.0000 | 152.0000 |
| | Weighted Size | 2472.5000 | 2472.5000 |
| | Total | 0.0000 | 0.0000 |
| | Lower 95% Limit Total | 0.0000 | 0.0000 |
| | Upper 95% Limit Total | 0.0000 | 0.0000 |
| | Mean | 0.0000 | 0.0000 |
| | SE Mean | 0.0000 | 0.0000 |
| | Lower 95% Limit Mean | 0.0000 | 0.0000 |
| | Upper 95% Limit Mean | 0.0000 | 0.0000 |
| M100 dummy-diff @candidate2 1=yes 0=no | Sample Size | 165.0000 | 165.0000 |
| | Weighted Size | 2597.5000 | 2597.5000 |
| | Total | 0.0000 | 0.0000 |
| | Lower 95% Limit Total | 0.0000 | 0.0000 |
| | Upper 95% Limit Total | 0.0000 | 0.0000 |
| | Mean | 0.0000 | 0.0000 |
| | SE Mean | 0.0000 | 0.0000 |
| | Lower 95% Limit Mean | 0.0000 | 0.0000 |
| | Upper 95% Limit Mean | 0.0000 | 0.0000 |



Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| M100 dummy-diff @candidate3 1=yes 0=no | Sample Size | 165.0000 | 165.0000 |
| | Weighted Size | 2597.5000 | 2597.5000 |
| | Total | 14.5000 | 14.5000 |
| | Lower 95% Limit Total | -13.2417 | -13.2417 |
| | Upper 95% Limit Total | 42.2417 | 42.2417 |
| | Mean | 0.0056 | 0.0056 |
| | SE Mean | 0.0054 | 0.0054 |
| | Lower 95% Limit Mean | -0.0051 | -0.0051 |
| | Upper 95% Limit Mean | 0.0163 | 0.0163 |
| M100 dummy-diff @candidate4 1=yes 0=no | Sample Size | 89.0000 | 89.0000 |
| | Weighted Size | 1500.5000 | 1500.5000 |
| | Total | 0.0000 | 0.0000 |
| | Lower 95% Limit Total | 0.0000 | 0.0000 |
| | Upper 95% Limit Total | 0.0000 | 0.0000 |
| | Mean | 0.0000 | 0.0000 |
| | SE Mean | 0.0000 | 0.0000 |
| | Lower 95% Limit Mean | 0.0000 | 0.0000 |
| | Upper 95% Limit Mean | 0.0000 | 0.0000 |



Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| iVo dummy-diff @candidate1 1=yes 0=no | Sample Size | 45.0000 | 45.0000 |
| | Weighted Size | 734.0000 | 734.0000 |
| | Total | 0.0000 | 0.0000 |

| | | | |
|---|---|---|---|
| | Lower 95% Limit | | |
| | Total | 0.0000 | 0.0000 |
| | Upper 95% Limit | | |
| | Total | 0.0000 | 0.0000 |
| | Mean | 0.0000 | 0.0000 |
| | SE Mean | 0.0000 | 0.0000 |
| | Lower 95% Limit | | |
| | Mean | 0.0000 | 0.0000 |
| | Upper 95% Limit | | |
| | Mean | 0.0000 | 0.0000 |
| | | | |
| iVo dummy-diff | Sample Size | 46.0000 | 46.0000 |
| @candidate2 | Weighted Size | 747.5000 | 747.5000 |
| 1=yes 0=no | Total | 10.0000 | 10.0000 |
| | Lower 95% Limit | | |
| | Total | -8.8106 | -8.8106 |
| | Upper 95% Limit | | |
| | Total | 28.8106 | 28.8106 |
| | Mean | 0.0134 | 0.0134 |
| | SE Mean | 0.0127 | 0.0127 |
| | Lower 95% Limit | | |
| | Mean | -0.0118 | -0.0118 |
| | Upper 95% Limit | | |
| | Mean | 0.0386 | 0.0386 |



Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| | | | |
| iVo dummy-diff | Sample Size | 46.0000 | 46.0000 |
| @candidate3 | Weighted Size | 747.5000 | 747.5000 |
| 1=yes 0=no | Total | 23.0000 | 23.0000 |
| | Lower 95% Limit | | |
| | Total | -8.0992 | -8.0992 |
| | Upper 95% Limit | | |
| | Total | 54.0992 | 54.0992 |
| | Mean | 0.0308 | 0.0308 |
| | SE Mean | 0.0210 | 0.0210 |
| | Lower 95% Limit | | |
| | Mean | -0.0109 | -0.0109 |
| | Upper 95% Limit | | |
| | Mean | 0.0724 | 0.0724 |

| | | | |
|---|---|---|---|
| iVo dummy-diff @candidate4 1=yes 0=no | Sample Size | 30.0000 | 30.0000 |
| | Weighted Size | 522.5000 | 522.5000 |
| | Total | 0.0000 | 0.0000 |
| | Lower 95% Limit Total | 0.0000 | 0.0000 |
| | Upper 95% Limit Total | 0.0000 | 0.0000 |
| | Mean | 0.0000 | 0.0000 |
| | SE Mean | 0.0000 | 0.0000 |
| | Lower 95% Limit Mean | 0.0000 | 0.0000 |
| | Upper 95% Limit Mean | 0.0000 | 0.0000 |

# Attachment B.4



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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method, Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105



Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| Total dummy-diff @precin 1=yes 0=no | Sample Size | 205.0000 | 205.0000 |
| | Weighted Size | 3222.0000 | 3222.0000 |
| | Total | 37.5000 | 37.5000 |
| | Lower 95% Limit Total | -4.1745 | -4.1745 |
| | Upper 95% Limit Total | 79.1745 | 79.1745 |
| | Mean | 0.0116 | 0.0116 |
| | SE Mean | 0.0065 | 0.0065 |
| | Lower 95% Limit Mean | -0.0013 | -0.0013 |
| | Upper 95% Limit Mean | 0.0246 | 0.0246 |
| M100 dummy-diff @precin 1=yes | Sample Size | 165.0000 | 165.0000 |
| | Weighted Size | 2597.5000 | 2597.5000 |

| | | | |
|---|---|---|---|
| 0=no | Total | 14.5000 | 14.5000 |
| | Lower 95% Limit | | |
| | Total | -13.2417 | -13.2417 |
| | Upper 95% Limit | | |
| | Total | 42.2417 | 42.2417 |
| | Mean | 0.0056 | 0.0056 |
| | SE Mean | 0.0054 | 0.0054 |
| | Lower 95% Limit | | |
| | Mean | -0.0051 | -0.0051 |
| | Upper 95% Limit | | |
| | Mean | 0.0163 | 0.0163 |



Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| iVo dummy-diff | Sample Size | 46.0000 | 46.0000 |
| @precin 1=yes | Weighted Size | 747.5000 | 747.5000 |
| 0=no | Total | 23.0000 | 23.0000 |
| | Lower 95% Limit | | |
| | Total | -8.0992 | -8.0992 |
| | Upper 95% Limit | | |
| | Total | 54.0992 | 54.0992 |
| | Mean | 0.0308 | 0.0308 |
| | SE Mean | 0.0210 | 0.0210 |
| | Lower 95% Limit | | |
| | Mean | -0.0109 | -0.0109 |
| | Upper 95% Limit | | |
| | Mean | 0.0724 | 0.0724 |

# Attachment B.5



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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method, Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105

Date: 05-18-2013 SUDAAN Page: 1
Time: 10:25:54 Table: 1

Frequencies and Values for CLASS Variables
by: M100.

| M100 | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 40 | 0 |
| Ordered Position: 2 | 165 | 1 |

Date: 05-18-2013 SUDAAN Page: 2
Time: 10:25:54 Table: 1

Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, M100.

| Variable | Sample | Weighted | Weighted | Weighted |
|---|---|---|---|---|

| M100 | Size | Size | X-Sum | Y-Sum | Ratio Est. | SE Ratio |
|---|---|---|---|---|---|---|
| UNDERVOTE/total counts for all candidates @precinct | | | | | | |
| Total | 205 | 3222 | 3232622 | 15 | 0.00000 | 0.00000 |
| 0 | 40 | 625 | 693917 | 0 | 0.00000 | 0.00000 |
| 1 | 165 | 2598 | 2538706 | 15 | 0.00001 | 0.00001 |

Proportion of overvotes in total M100 votes



09:41 Saturday, May 18, 2013

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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method, Assuming a Stratified, Without Replacement (STRWOR) Design

Sample Weight: SAMPLINGWEIGHT

Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222

Denominator degrees of freedom : 105



Frequencies and Values for CLASS Variables

by: M100.

| M100 | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 40 | 0 |
| Ordered Position: 2 | 165 | 1 |

Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, M100.

| Variable M100 | Sample Size | Weighted Size | Weighted X-Sum | Weighted Y-Sum | Ratio Est. | SE Ratio |
|---|---|---|---|---|---|---|
| OVERVOTE/total counts for all candidates @precinct | | | | | | |
| Total | 205 | 3222 | 3232622 | 154 | 0.00005 | 0.00003 |
| 0 | 40 | 625 | 693917 | 154 | 0.00022 | 0.00017 |
| 1 | 165 | 2598 | 2538706 | 0 | 0.00000 | 0.00000 |



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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method, Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105



Frequencies and Values for CLASS Variables
by: IVOTRONIC.

| IVOTRONIC | Frequency | Value |
|---|---|---|
| Ordered Position: 1 | 159 | 0 |
| Ordered Position: 2 | 46 | 1 |

---

Date: 05-18-2013 SUDAAN Page: 2
Time: 10:26:39 Table: 1

Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, IVOTRONIC.

| Variable IVOTRONIC | Sample Size | Weighted Size | Weighted X-Sum | Weighted Y-Sum | Ratio Est. | SE Ratio |
|---|---|---|---|---|---|---|
| UNDERVOTE/total counts for all candidates @precinct | | | | | | |
| Total | 205 | 3222 | 3232622 | 15 | 0.00000 | 0.00000 |
| 0 | 159 | 2475 | 2466102 | 15 | 0.00001 | 0.00001 |
| 1 | 46 | 748 | 766521 | 0 | 0.00000 | 0.00000 |

Proportion of overvotes in total iVotronic votes 70
09:41 Saturday, May 18, 2013

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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method, Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105

Date: 05-18-2013 SUDAAN Page: 1
Time: 10:26:39 Table: 1

Frequencies and Values for CLASS Variables
by: IVOTRONIC.

| IVOTRONIC | Frequency | Value |
|---|---|---|

| Ordered Position: | | |
|---|---|---|
| 1 | 159 | 0 |
| Ordered Position: | | |
| 2 | 46 | 1 |



Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, IVOTRONIC.

| Variable IVOTRONIC | Sample Size | Weighted Size | Weighted X-Sum | Weighted Y-Sum | Ratio Est. | SE Ratio |
|---|---|---|---|---|---|---|
| OVERVOTE/total counts for all candidates @precinct | | | | | | |
| Total | 205 | 3222 | 3232622 | 154 | 0.00005 | 0.00003 |
| 0 | 159 | 2475 | 2466102 | 0 | 0.00000 | 0.00000 |
| 1 | 46 | 748 | 766521 | 154 | 0.00020 | 0.00015 |

# Attachment C.1

# Attachment C.2

# Attachment C.1

Mean Discrepancy of the Vote Count for Romney

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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method,
Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105

Date: 05-05-2013 SUDAAN Page: 1
Time: 13:53:40 Table: 1

Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| ERR_OBAMA | Sample Size | 204.0000 | 204.0000 |
| | Weighted Size | 3197.0000 | 3197.0000 |
| | Mean | 0.0885 | 0.0885 |
| | SE Mean | 0.0429 | 0.0429 |
| | DEFF Mean #4 | 1.2964 | 1.2964 |

H-T Estimation of A total (Actual Vote Combined for Obama and Romney)

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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method,
Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105

Date: 05-05-2013 SUDAAN Page: 1
Time: 14:01:29 Table: 1

Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| TOTRECOUNTWL | Sample Size | 205.0000 | 205.0000 |
| | Weighted Size | 3222.0000 | 3222.0000 |
| | Total | 3079818.5000 | 3079818.5000 |
| | SE Total | 424511.4725 | 424511.4725 |

PROC RATIO Step for Estimating Total A

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DESIGN SUMMARY: Variances will be computed using the Taylor Linearization Method,
Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105

Date: 05-05-2013 SUDAAN Page: 1
Time: 14:02:17 Table: 1

Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| TOTRECOUNTWL/T-OTCOUNTWL | Ratio Est. | 1.018678 | 1.018678 |
| | SE Ratio | 0.003000 | 0.003000 |
| | Weighted Y-Sum | 3079819 | 3079819 |
| | Weighted X-Sum | 3023347 | 3023347 |

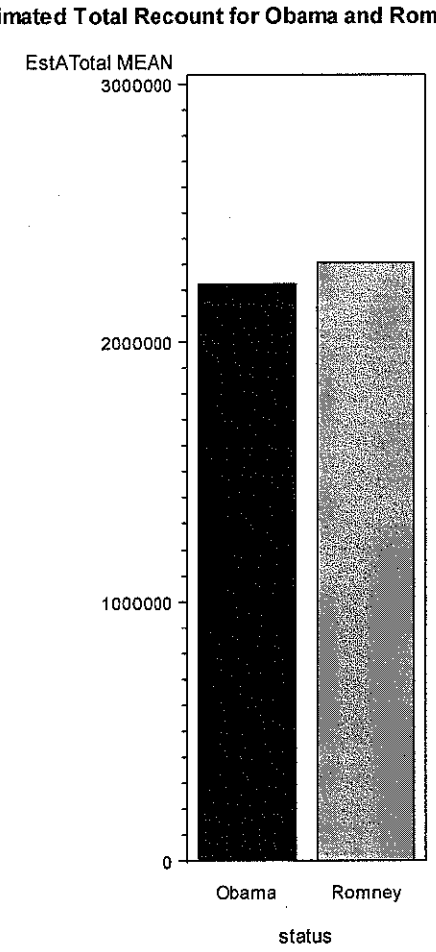
Probability that Obama defeated Romney

| Obs | MEAN | SEMEAN | WSUM | ATOTAL | SEATOTAL |
|---|---|---|---|---|---|
| 1 | -0.07 | 0.12 | 3222.00 | 3079818.50 | 424511.47 |
| 2 | -0.07 | 0.12 | 3222.00 | 3079818.50 | 424511.47 |
| 3 | -0.07 | 0.12 | 3222.00 | 3079818.50 | 424511.47 |

| Obs | RatioA | SERatioA | Dzero | t | probt | DiffProb |
|---|---|---|---|---|---|---|
| 1 | 4531882.52 | 13345.92 | 14.277 | 122.97 | 1 | 3.9824E-112 |
| 2 | 4531882.52 | 13345.92 | 1.382 | 12.45 | 1 | 1.97951E-22 |
| 3 | 4531882.52 | 13345.92 | 355.837 | 3050.37 | 1 | 7.8706E-253 |

# Attachment C.2

**Figure 1. Estimate of Actual Statewide Vote Count for Election Day Obama and Romney**



Estimating ratio for A_Romney and A_Obama

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Assuming a Stratified, Without Replacement (STRWOR) Design
Sample Weight: SAMPLINGWEIGHT
Stage 1 Stratification Variable: CID

Number of observations read : 205 Weighted count : 3222
Denominator degrees of freedom : 105

Date: 05-05-2013 SUDAAN Page: 1
Time: 14:21:02 Table: 1

Variance Estimation Method: Taylor Series (STRWOR)
by: Variable, SUDAAN Reserved Variable One.

| Variable | | SUDAAN Reserved Variable One | |
|---|---|---|---|
| | | Total | 1 |
| REC_ROMNEY/M_ROMNEY | Ratio Est. | 1.016772 | 1.016772 |
| | SE Ratio | 0.002977 | 0.002977 |
| | Weighted Y-Sum | 1574011 | 1574011 |
| | Weighted X-Sum | 1548047 | 1548047 |
| REC_OBAMA/M_OBAMA | Ratio Est. | 1.020679 | 1.020679 |
| | SE Ratio | 0.006001 | 0.006001 |
| | Weighted Y-Sum | 1505808 | 1505808 |
| | Weighted X-Sum | 1475300 | 1475300 |

95% Confidence Interval for the Estimated Actual Total Recount of Romney and Obama

| Obs | status | RHAT | SERHAT | WYSUM | WXSUM |
|---|---|---|---|---|---|
| 1 | Romney | 1.02 | 0.00 | 1574011.00 | 1548047.00 |
| 2 | Obama | 1.02 | 0.01 | 1505807.50 | 1475300.00 |

| Obs | Vtotal | seATotal | EstATotal | UCI95 | LCI95 |
|---|---|---|---|---|---|
| 1 | 2270395 | 6758.58 | 2308474.29 | 2321721.11 | 2295227.48 |
| 2 | 2178391 | 13071.51 | 2223437.61 | 2249057.77 | 2197817.45 |

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